Dairy herd renewal policy: economic calculations and optimization

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Summary

The crucial problem in managing the dairy herd renewal processes, as well as its size, is the optimizing the timeframe of economic use of animals, depending on the prospects for the development of production and market conditions. The aim of the study was to evaluate and improve methods for calculating the optimal duration of economic use of dairy cows. Models with and without taking into account stochastic characteristics of the studied process were used for the analysis. The results of the survey conducted in 2018-2019 in LLP "Olzhasadchikovskoe" located in the northern part of Kazakhstan were used as initial data. It is noted that the greatest difficulties in solving the problem arise while optimizing the plan for renewing the existing herd at the expense of animals with high productive potential. This is explained mainly by the fact that animals of different productive ages have different economic value due to changes in their milk productivity with age with a certain regularity. The optimal duration of productive life of dairy cows with an average annual yield of 5,000 – 6,000 kg per head in the Northern Kazakhstan is about six years. It is noted that in Kazakhstan the widespread use of calculation methods with and without stochastic characteristics of dairy production is limited by the fact that in most agricultural enterprises there is no accounting of data relating to the patterns of change in the economic value of animals depending on their age, as well as probabilistic characteristics of milk productivity (cow mortality rate, infertility, etc.). The key issue in the development of an effective strategy of dairy herd management is the forecasting of price dynamics for milk and feed. In addition, in developing economies, including Kazakhstan, effective management of dairy herd renewal processes requires, first of all, the organization and implementation of management accounting system in animal husbandry.

Key words

dairy cattle breeding, productive life expectancy, lactation, marginal income, herd renewal, planning, decision-making

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Introduction

Investment decisions in the livestock economy ultimately focus on the problem of managing the reproduction of the main herd. The reproduction can be simple or extended depending on the economic goals of the entrepreneur. In the process of herd management and its renewal, the key issue is the optimization of the timeframe of use of cows. This issue is one of the most important for cattle breeders because of the existence of a certain pattern in changing the productive potential of cows as they age: first, there is an increase in the productivity of the animal, then after three or four years of stabilization there is a steady decline (Kostomahin, 2009).

The issue of optimization of herd reproduction management in cattle breeding allows several variants of formulation, each of which has its own specifics and, accordingly, requires a special methodological approach for its solution. Of the tasks of optimizing the strategy of herd renewal, the most common is the one in which the culling of the animal implies its replacement by another having a similar economic value. This is a case of simple reproduction of the herd. In another variant, the task is to determine the optimal strategy for renewing the herd at the expense of animals with improved economic characteristics. In both cases, the possibility of varying the size of the herd for economic reasons is not excluded: culling an animal does not necessarily mean replacing it with another, and, conversely, the introduction of a new animal into the herd does not always lead to the removal of the "old" one. The economic rationale for increasing or decreasing production cannot ignore the fact that the unit cost of production is a function of the herd size. Moreover, this function is nonlinear and graphically represents a U-shaped curve. That is, at first the cost per unit of production decreases as the production volume increases, then, reaching a certain minimum level, again increases. It means that the decision to expand or reduce the size of the herd, taken in the t-th year, to some extent predetermines the nature of the decision on a similar issue in a year t+1, t+2, ... . Each previous decision affects subsequent decisions by changing the cost of production. Therefore, making a decision on the economic feasibility of increasing or reducing the size of the herd requires taking into account this dependence. The defining problem in managing the dairy herd renewal processes, as well as its size, is the question of accounting both the time factor and the differences in the life cycles of their cycles, a different approach to solving the problem is required. There is a need for a methodology for calculating and comparing alternative capital investment plans that would take into account both the time factor and the differences in the life cycle of the options under consideration, and the infinity of the planning horizon.

Comparing the P(T) of different variants of investment plans based on formula (1) to choose the best of them makes sense if the projects with the same life cycle are considered. However, when we are faced with options that have differences in the length of their cycles, a different approach to solving the problem is required. There is a need for a methodology for calculating and comparing alternative capital investment plans that would take into account both the time factor and the differences in the life cycle of the options under consideration, and the infinity of the planning horizon.

In relation to the conditions of our task, these requirements are satisfied by the methodology the implementation of which involves the calculations in two stages: (a) using the formula (1) we calculate the total discounted marginal income P(T) from the use of animals during the T years of productive life, T=1,2,..., and (b) for each variant of the duration of economic use of a cow the average annual marginal income P_{average}(T) is calculated with the formula

\[ P_{average}(T) = P(T)[r/(1-1/(1+r)^T)], \quad T = 1,2, \ldots \]  

However, the optimal duration of economic use of the animal will not answer a question about optimizing a plan to renew the existing herd by animals with a more productive potential. This is explained mainly by the fact that animals of different ages have different economic value due to changes in their milk productivity as they mature and age. Therefore, the answer to the question of replacing an animal from the existing herd is always individual and requires taking into account its current and expected productivity in subsequent years.

Methods and data

The results of the survey conducted in 2018 and 2019 on milk production in LLP "Olzha-Sadchikovskoe" located in Kostanay region of Kazakhstan were used as the initial data. Data from Table 1 show that cows after their sixth calving have sharply declining economic value in all respects. As a consequence, farm income from cow use also tends to vary by age with the same pattern.

When calculating the optimum duration of the maintenance of an animal in the herd the unlimited planning period is accepted. This is not contrary to common sense, because the demand for food products, including meat and milk, is in principle eternal.

To calculate the total income for T years of keeping an animal, taking into account the time factor (the amount of discounted marginal income), the formula has been used that is a modification of a formula proposed by Perrin to calculate the optimal duration of the use of production capital (Perrin, 1972):

\[ P(T) = \sum_{t=1}^{T} (1+r)^{-t} [D(t) - Z(t)] + (1+r)^{-T} S(T) - S(0) \]

where P(T) is the discounted marginal income (MI) from a cow for T years, tenge; D(t) – revenue from a cow in year t of lactation, tenge; Z(t) – the cost of maintaining a cow in the herd in year t of lactation, tenge; S(T) – revenue from the sale of a cow after T years of its stay in the herd, tenge; S(0) - cow replacement costs, tenge; r – discount rate; t – current productive age of a cow, year.

Comparing the P(T) of different variants of investment plans based on formula (1) to choose the best of them makes sense if the projects with the same life cycle are considered. However, when we are faced with options that have differences in the length of their cycles, a different approach to solving the problem is required. There is a need for a methodology for calculating and comparing alternative capital investment plans that would take into account both the time factor and the differences in the life cycle of the options under consideration, and the infinity of the planning horizon.

In relation to the conditions of our task, these requirements are satisfied by the methodology the implementation of which involves the calculations in two stages: (a) using the formula (1) we calculate the total discounted marginal income P(T) from the use of animals during the T years of productive life, T=1,2,..., and (b) for each variant of the duration of economic use of a cow the average annual marginal income P_{average}(T) is calculated with the formula

\[ P_{average}(T) = P(T)[r/(1-1/(1+r)^T)], \quad T = 1,2, \ldots \]
As in the conditions of the considered problem the minimum necessary rate of return $r$ on investment is identical for all possible options, the best of them will be that which provides the maximum level of average annual marginal income $P_{\text{average}}(T)$ from economic use of animals. Keeping an animal in herds for more or less years, at which the maximum average annual income is achieved, will lead to a decrease in the value of the indicator under consideration.

Methodically, a more correct solution of the problem involves taking into account the stochastic characteristics of the process under study: the probability distribution of mortality, unplanned culling of animals, offsprings per 100 cows for years of productive life, and others. In this case the calculation formula takes a more complex form and the task will be to maximize the value of the function

$$P_{\text{average}}(T) = \sum_{t=0}^{T} \frac{(1+r)^{-t} \cdot B(t) \cdot p(t-1) + (1+r) \cdot S(T) \cdot 1 - p(t) \cdot (1+r)^{-t} \cdot p(t) \cdot S(0)}{(1+r)^{T}}$$

where $P_{\text{average}}(T)$ – the average annual marginal income from a cow over $T$ years of productive life, tenge; $ER(t)$ – expected marginal income from a cow in year $t$, tenge; $p(t-1)$ – probability that a cow of productive age $(t-1)$ will move to the next age group of $t$. Here, $p(0) = 1$; $p(t)$ - probability that a cow of productive age $t$ will move to the next age group of $(t+1)$; $p_c(T)$ – probability of death of a cow at the age of $T$; $S(T)$ – market value of a cow at the end of $T$ years of productive life, tenge; $S(0)$ - market value of heifer being introduced into the herd, tenge; $r$ – discount rate.

And, $ER(t) = P(t)\cdot p(t) + R(t)\cdot p(t) - S(0)\cdot (1-p(t))$

where $P(t)$ – marginal income from a cow of the age of $t$, tenge; $R(t)$ – sale revenue from a cow of age $t$ in the case of production culling (infertility, illness, low productivity), tenge; $S(0)$ – cow replacement costs, tenge/head; $P(t)$ – probability of unplanned culling of a cow due to production defects in year $t$ of productive life; $t$ – current productive age of a cow, years.

Calculations according to the scheme were carried out using the data of LLP "Olzha-Sadchikovskoe" located in Kostanay region of Kazakhstan. On the farm the dairy herd consisted of cows of Holstein breed. Data on milk productivity (commodity part) of cows by years of lactation, revenue from production and sale of milk, revenue from the sale of culled animals and the cost of their maintenance per head are shown in Table 2. The cost for replacing the cows with heifers was taken as 265,000 tenge per head. Milk was sold at the price of 100 tenge per litre, and cattle for meat at the price of 540 - 560 tenge per kg of live weight. The discount rate is assumed to be 0.05.

As follows from Table 2, in the ninth year of the productive life of a cow, the annual revenue from its use becomes less than the total annual cost of its maintenance. In other words, keeping an animal for more than eight years in the herd is economically unjustified under any circumstances. Therefore, to solve the problem, it suffices to use data relating to the first eight years of a cow in the herd. The average annual volume of commercial milk yield for eight years is 5,918 kg per head.

### Results and Discussion

The calculations were carried out using information and analytical system "Agro Optim" developed in Seifullin Kazakh Agrotechnical University. Table 3 shows the results of calculations of economic efficiency of different terms of productive life of dairy cows in the herd without taking into account the stochastic characteristics of the process.

As follows from Table 2, in the ninth year of the productive life of a cow, the annual revenue from its use becomes less than the total annual cost of its maintenance. In other words, keeping an animal for more than eight years in the herd is economically unjustified under any circumstances. Therefore, to solve the problem, it suffices to use data relating to the first eight years of a cow in the herd. The average annual volume of commercial milk yield for eight years is 5,918 kg per head.

The optimal duration of productive life of a cow on the farm (calculated without taking into account the stochastic characteristics) is five to six years. And then, the highest average annual discounted marginal income of slightly more than 341,000 tenge per head is achieved.

Table 4 shows the results of calculations of the economic efficiency of the timeframe of the animal in the dairy herd, taking into account the probabilistic characteristics.

The optimum duration of a cow in the dairy herd on the farm with probabilistic characteristics taken into account is six years. And then, the expected size of the average annual marginal income will be equal to slightly less than 292,000 tenge per head.

It is obvious that the account of stochastic elements in the considered problem leads to a dramatic increase of amount of calculations in need. However, with the development of information technologies in the industry, the problem of automation of calculations loses its relevance. The major factor limiting the widespread use of calculation methods based on formulas (3) and (4), in particular in Kazakhstan, is the fact that on most farms, large and small, the accounting of data related to the patterns of change in the economic value of animals depending on their age, as well as the probabilistic characteristics of the process, is not well established and developed. Apart is the question of forecasting the movement of market prices for milk and feed.

### Table 1. Relationship between the economic value and productive age of cows

<table>
<thead>
<tr>
<th>Productive age of cow, years</th>
<th>Offspring per 100 cows</th>
<th>Mortality rate of cattle, %</th>
<th>Milk yield, kg/head</th>
<th>Productive age of cow, years</th>
<th>Offspring per 100 cows</th>
<th>Mortality rate of cattle, %</th>
<th>Milk yield, kg/head</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85.5</td>
<td>2.25</td>
<td>5,400</td>
<td>6</td>
<td>93.0</td>
<td>2.80</td>
<td>6,513</td>
</tr>
<tr>
<td>2</td>
<td>89.0</td>
<td>2.25</td>
<td>6,083</td>
<td>7</td>
<td>90.8</td>
<td>3.25</td>
<td>6,178</td>
</tr>
<tr>
<td>3</td>
<td>92.7</td>
<td>2.30</td>
<td>6,540</td>
<td>8</td>
<td>87.0</td>
<td>3.70</td>
<td>5,754</td>
</tr>
<tr>
<td>4</td>
<td>94.5</td>
<td>2.35</td>
<td>6,698</td>
<td>9</td>
<td>82.0</td>
<td>4.35</td>
<td>5,375</td>
</tr>
<tr>
<td>5</td>
<td>94.3</td>
<td>2.45</td>
<td>6,671</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Distribution of milk yields (commodity part), revenues from the sale of milk and culled cows, and the cost of their stay in the herd by years of productive life

<table>
<thead>
<tr>
<th>Productive age, years</th>
<th>Milk yield (commodity part), kg/head</th>
<th>Revenue, tenge/head</th>
<th>Total annual cost of maintenance, tenge/head</th>
<th>Annual amount of maintenance variable costs, tenge/head</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>From use</td>
<td>From culling</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5,130</td>
<td>513,000</td>
<td>322,000</td>
<td>532,450</td>
</tr>
<tr>
<td>2</td>
<td>5,779</td>
<td>577,876</td>
<td>346,150</td>
<td>534,730</td>
</tr>
<tr>
<td>3</td>
<td>6,213</td>
<td>621,327</td>
<td>371,194</td>
<td>535,500</td>
</tr>
<tr>
<td>4</td>
<td>6,363</td>
<td>636,344</td>
<td>371,194</td>
<td>536,125</td>
</tr>
<tr>
<td>5</td>
<td>6,337</td>
<td>633,741</td>
<td>364,566</td>
<td>536,800</td>
</tr>
<tr>
<td>6</td>
<td>6,187</td>
<td>618,724</td>
<td>364,566</td>
<td>536,278</td>
</tr>
<tr>
<td>7</td>
<td>5,869</td>
<td>586,886</td>
<td>357,938</td>
<td>536,100</td>
</tr>
<tr>
<td>8</td>
<td>5,466</td>
<td>546,639</td>
<td>357,938</td>
<td>535,478</td>
</tr>
<tr>
<td>9</td>
<td>5,106</td>
<td>510,597</td>
<td>357,938</td>
<td>535,400</td>
</tr>
</tbody>
</table>

Table 3. Economic efficiency of a dairy cow by years of lactation (probabilistic characteristics not taken into account)

<table>
<thead>
<tr>
<th>Year of productive life</th>
<th>The cumulative MI from the milk production discounted, tenge per head</th>
<th>Revenue from the sale of a cow discounted, tenge per head</th>
<th>Total discounted income, tenge per head</th>
<th>Average annual MI discounted, tenge per head</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>245,166</td>
<td>306,667</td>
<td>286,174</td>
<td>300,483</td>
</tr>
<tr>
<td>2</td>
<td>536,531</td>
<td>313,905</td>
<td>584,778</td>
<td>314,496</td>
</tr>
<tr>
<td>3</td>
<td>851,192</td>
<td>320,726</td>
<td>906,260</td>
<td>332,786</td>
</tr>
<tr>
<td>4</td>
<td>1,162,964</td>
<td>305,453</td>
<td>1,202,759</td>
<td>339,192</td>
</tr>
<tr>
<td>5</td>
<td>1,457,598</td>
<td>285,713</td>
<td>1,477,852</td>
<td>341,300</td>
</tr>
<tr>
<td>6</td>
<td>1,727,196</td>
<td>272,107</td>
<td>1,733,945</td>
<td>341,558</td>
</tr>
<tr>
<td>7</td>
<td>1,961,416</td>
<td>254,438</td>
<td>1,950,196</td>
<td>337,033</td>
</tr>
<tr>
<td>8</td>
<td>2,157,409</td>
<td>242,322</td>
<td>2,134,707</td>
<td>330,188</td>
</tr>
</tbody>
</table>

Table 4. Economic efficiency of productive life of a dairy cow (with probabilistic characteristics taken into account)

<table>
<thead>
<tr>
<th>Year of productive life</th>
<th>The cumulative MI from the milk production discounted, tenge per head</th>
<th>Revenue from the sale of a cow discounted, tenge per head</th>
<th>Total discounted income, tenge per head</th>
<th>Average annual MI discounted, tenge per head</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210,497</td>
<td>299,767</td>
<td>245,264</td>
<td>257,527</td>
</tr>
<tr>
<td>2</td>
<td>471,401</td>
<td>262,887</td>
<td>469,288</td>
<td>252,385</td>
</tr>
<tr>
<td>3</td>
<td>762,376</td>
<td>238,443</td>
<td>735,819</td>
<td>270,199</td>
</tr>
<tr>
<td>4</td>
<td>1,054,601</td>
<td>210,403</td>
<td>1,000,004</td>
<td>282,013</td>
</tr>
<tr>
<td>5</td>
<td>1,329,862</td>
<td>185,791</td>
<td>1,250,653</td>
<td>288,869</td>
</tr>
<tr>
<td>6</td>
<td>1,578,140</td>
<td>166,259</td>
<td>1,479,399</td>
<td>291,467</td>
</tr>
<tr>
<td>7</td>
<td>1,788,582</td>
<td>143,911</td>
<td>1,667,493</td>
<td>288,176</td>
</tr>
<tr>
<td>8</td>
<td>1,958,257</td>
<td>123,870</td>
<td>1,705,644</td>
<td>263,900</td>
</tr>
</tbody>
</table>
In the context of the problem under consideration, the peculiarities of the herd size change over time, productivity of dairy cows, and efficiency of dairy sector in countries with developed economies are of interest. The average size of dairy herds has continuously increased over recent decades in all developed countries. Concurrently, the number of dairy farms has decreased in most countries. In the USA, the number of dairy farms decreased from 139,670 in 1995 to 49,331 in 2012 (USDA-NASS, 1999, 2013a). However, since 1995, the total US dairy herd decreased by only 2.5% (USDA-NASS, 1999, 2013b). Consequently, cows are increasingly managed in fewer, albeit larger herds. In 2012, the 32% of the US dairy herds with less than 30 cows made 1.6% of the national herd, whereas the 1.3% of the US herds with more than 2000 cows made 33% of the national herd. The increase in herd size is driven by economies of scale; the cost of production per unit decreases with an increasing herd size (Wilson, 2011; Wolf, 2013). The increase in average herd size has been less pronounced in countries with a quota system (EU until March 2015 and Canada). The increase in milk production in Canada has been achieved almost exclusively through increased milk production per cow, with cow numbers declining by 13% since 2000. Concurrently, the average Canadian dairy herd increased from 52 cows in 1996 to 79 cows in 2014, again reflecting a reduction in the number of farms. Mortality rates in US dairy herds increased with increasing herd size (Shahid et al., 2015). Increased milk production is often associated with decreased health of dairy cows (Koeck et al., 2014).

The profitability of dairy production is highly dependent on milk price. In most countries with a developed dairy industry, except those with a quota system, the price paid to producers for milk is not regulated and consequently can be highly volatile, even over short intervals. Over the last two decades, many developed countries, such as Australia, Switzerland, and countries within the European Union, have abandoned supply management (quota) systems; consequently, given market pressures, these countries have often experienced fluctuating (typically declining) milk prices to align with global prices (Sinclair et al., 2014). Decreases in milk price result in reduced profitability and ultimately lead to abrupt herd size increase in an attempt to maintain cash flow. Sharp herd size increase may result in overstocking, thereby reducing access to primary resources by individual cows, which may diminish health or performance - of individual cows and eventually of the group or herd (von Keyserlingk and Weary, 2010).

The given data and results of researches testify that questions of optimization of strategy of herd renewal and effective management of its size are of paramount importance for ensuring sustainable development of the sector and, thus, do not lose the relevance.

The results of the first studies on the optimization of the herd renewal strategy have value in the conditions of static state of external factors, primarily the prices for products and production resources (Jarvis, 1974). In addition, there in research have been excluded such important stochastic characteristics of animal production, as animals' mortality rate, infertility and others, from the analysis. Melton (Melton, 1980) considered the effect of improving the genetic productive potential of animals and concluded that the overall increase in livestock productivity leads to a reduction of the optimal age for culling.

Of the other published works, the articles by King (1979), Bentley and Shumway (1981), and Schmitz (1997) should first be alluded to. The common and most important thing in these works is that they recognize the economic feasibility and the principal possibility of varying the size of the herd, taking into account market conditions. The "adaptive" herd size management algorithms that they use are relevant to herd size management in beef cattle, considering the product price cycles. Trapp (1986), who also analysed the economics of beef cattle breeding, considered separately the decisions on culling "old" and entering "new" animals into the herd. Thus, it is assumed that for economic reasons, culling an animal does not necessarily mean replacing it with another, and conversely, the introduction of a heifer into the herd does not always lead to the sale or slaughter of an "old" cow. It should be noted that the price movement in the milk market has significant differences from similar processes in the meat market.

Beale et al. (1983) tell us sound judgments about what measures should be taken to minimize losses during periods of falling market prices for livestock products. Their main idea is that during periods of deterioration of market conditions farmers should: cull adult animals as much as possible; have alternative areas to use the released resources; practice hedging. Unfortunately, the authors of the publication do not express their recommendations in the form of formulas that allow to quantify the effectiveness of decisions made on their basis.

Chavas and Klemme (1986) note that the dynamics of the dairy herd population has a decisive influence on the process of economic adaptation of milk production to changes in market prices for the product. The steady rise in milk prices provides economic incentives to increase production. At the same time, the coefficient of herd renewal, the main control variable, varies depending on the economic situation (prices for main and by-products, feed) and the productivity of animals of different age categories. The increase can be achieved both by expanding the herd and by increasing the productivity of cows. It is noted that farmers respond to favourable milk prices by rapidly replacing less productive animals with their more productive breeds, and this does not necessarily lead to an increase in herd size.

None of the works considers such a factor as the necessity for farmers to arrange payments on bank loans. Meanwhile, it is obvious that the decisions taken by farmers on production management in livestock industry are significantly influenced not only by the expected market prices for products, but also by the previously agreed debt repayment schedule (Bierlen et al., 1998).

The problem of optimization when farmers renew the herd with more productive (on average) animals occurs and has its own characteristics due to the fact that in certain years the economic life of the existing cows can have a productivity that exceeds the average annual potential of “new” animals with which it is planned to renew the herd. For example, in LLP "Olzha-Sadchikovskoe" there is an opportunity to update the dairy herd with animals whose average annual commercial milk yield is estimated at 6,175 kg per head. As follows from Table 2, in the 4th and 5th lactation, commodity productivity of available cows is slightly higher: 6363 and 6,337 kg per head, respectively. Therefore, it is not surprising that there are doubts about the economic feasibility of replacing animals whose age before the next lactation is three or four years.
The age $\tau$ of cows to be replaced by more productive animals, at the time of their possible culling, may be 1, 2, ..., years. If the cows of age $T - 1$, and needless to say $T$, clearly need replacement, then for animals of other ages the problem has no obvious solution. Therefore, it is quite reasonable to develop principles for optimizing the strategy of renewing the herd with animals with higher productive potential, taking into account the age distribution of cows to be replaced.

To solve this problem, it is important only to assess the economic value of the "old" cows in the following years of their possible use. For certainty, suppose the age of a cow under consideration equals 4, that is $\tau = 4$. Therefore, we need to take into account the economic value of the cow, starting from the 5th year of lactation. In general, this problem can be formulated as follows: to determine such a duration $X$ (number of years) of further staying of the "old" cow in the herd, in which the amount of income from it for additional years and the expected income from the use of the "new" animal will be maximum.

Let the average annual discounted income from the use of a new, more productive animal, to be estimated at $Q_\tau$. Then the procedure for calculating the optimal strategy for updating the herd will be as follows:

(a) define the duration $X^*$ of further staying of an "old" cow of age $\tau$ in the herd in which the average annual present value $Q(X^*)$ reaches its maximum level, that is

$Q(X^*) = \max \left\{ \sum_{x=1}^{N} \frac{1+r}{{(1+r)^x}} R(\tau + x) + (1+r)^{-x} S(\tau + x) - S(0) \right\}$

for $1 \leq x \leq N - \tau$, (5)

where $R(\tau + x) = D(\tau + x) - Z(\tau + x); N$ – maximum age of the animal at which the annual revenue from its use exceeds the total cost of its maintenance;

(b) if $Q(X^*) > Q_{\tau}$, then a cow of age $\tau$ should immediately be replaced by a "new" animal. Otherwise, an "old" cow should be kept in the herd. However, this does not mean that the optimal number of additional years of life of a given cow is equal to $X^*$. It may turn out that the increment $\Delta V(X)$ of the total income $V(X), X^* \leq X \leq N - \tau$, when using a cow in the year $(x+1)$, will be greater than $Q_\tau \cdot V(X)$ is calculated as

$V(X) = \sum_{x=1}^{X^*} \left( 1+r\right)^{-x} R(\tau + x) + (1+r)^{-x} S(\tau + x) - S(0)$

Formula (6) yields

$\Delta V(X) = V(X+1) - V(X) = (1+r)^{-x} \left[ R(\tau + X + 1) + (1+r)^{-1} S(\tau + X + 1) - S(\tau + x) \right]$ for $X^* \leq x \leq N - \tau$. (7)

Thus, when the condition $Q(X^*) > Q_{\tau}$ is true, the "old" cow of age $\tau$ should be used while $\Delta V(X) > Q_{\tau}, X^* \leq X < N - \tau$, that is, until the increment of the total income from the use of the "old" cow in year $X+1$ exceeds the expected annual income from the use of "new" animals with higher productive potential.

In the illustrated example of LLP “Ozha-Sadchikovskoe”, the expected average discounted marginal income from the use of animals with higher milk productivity is estimated as 367,445 tenge per head (the amount calculated using the rules (1) and (2)). For example, consider the case when the productive age of an "old" cow in the herd is four years, that is $\tau = 4$.

With the use of formula (5), we find $Q(X^*)$:

$Q(1) = [(1.05)^{-1}(633,741-257,664)+(1.05)^{-1}364,566-265,658] \times \left[ 0.05/\left(1-1/(1+0.05)^5\right) \right] = 461,702; \quad Q(2) = [(1.05)^{-1}(618,724-257,413)+(1.05)^{-1}364,566-265,658] \times \left[ 0.05/\left(1-1/(1+0.05)^5\right) \right] = 403,839.$

Since $Q(2) < Q(1)$, further calculations can be ceased.

Thus, $X^* = 1$ and $Q(X^*) = 461,702$. Due to the fact that $Q(X^*) > Q_{\tau}$, the cow should be kept in the herd. Now it is necessary to determine the optimal duration of its further economic use. On the basis of formulae (7) we find the value of the increment of income by lengthening the period of keeping a cow for each next year:

$\Delta V(1) = V(2) - V(1) = (1.05)^{-1} [(1+0.05)^{-1}(618,724-257,413) + (1.05) \times (364,566-265,658)] = 405,383; \quad \Delta V(3) = V(4) - V(3) = (1.05) \times (357,938-265,658) = 352,925.$

Since it turned out that $\Delta V(1) > Q_{\tau} > \Delta V(2)$, the cow, which by the beginning of the year has four lactations, should be used in addition for the next two years. The total duration of keeping this cow in the herd will make six years.

Summarizing, it should be noted that conceptual and methodological aspects of the analysis and adoption of optimal decisions on the dairy production management considering the volatility of market prices and rapid changes in animal productivity still require in-depth study and further development.

**Further research opportunities**

The use of different methodological approaches to the modelling of herd size management processes results in outcomes that differ from each other in the degree of accuracy. The results of most studies on the optimization of the herd renewal strategy are valuable in terms of relatively static state of external factors, primarily - the prices for products and resources. Research on optimization of managerial decisions on dairy herd in the conditions of the impermanent, changing with certain regularities, market prices seems to be perspective. In particular, the possible presence of cycles in the dynamics of market processes demands its up-close investigation. Rational variation of dairy herd is a crucial tool for the production processes effective adaptation to changes in the market prices. It is obvious that the coefficient of renewal of the herd as the main control variable will vary depending on the economic situation and productivity of animals. The factor that can create the greatest difficulties for the effective practical application of tools to optimize decisions on herd management is the problem of ensuring acceptable accuracy of price dynamics forecasts (for milk and, perhaps to a lesser extent, for production resources). Forecasting the price cycle remains a key issue in developing the effective market strategy.

**Concluding comments**

Calculations of the optimal duration of use of dairy cows in the herd, taking into account and without taking into account the stochastic characteristics, do not seem to produce different results. Therefore, in the absence of detailed data to describe the probabilistic properties of the process, it can be proposed
to carry out calculations without considering the probabilistic characteristics. But when deciding, one should not forget about the stochastic nature of the process and make appropriate adjustments.

Effective application of the presented methods for optimization of herd management decisions demands considerable preliminary work on preparation and systematization of necessary initial data. In other words, it requires the development and practice of appropriate management accounting system on farms.

The methodological techniques and procedures presented above can be used not only for the effective decision making on dairy farms, but also in any other livestock industry, including pigs, sheep, poultry, as well as in fruit growing, forestry. The key issue for the development of an effective dairy herd management strategy is the forecasting of price dynamics. In other words, the rational allocation of farm resources requires the price to be predicted with acceptable accuracy and reliability. Accurate short-term forecasts, if possible, whatsoever, require a detailed understanding of the relationship and interaction of the variables that determine supply and demand in the market. It should be noted that after decades of research with use of the most sophisticated statistical and econometric tools, few of the developed models have found any effective application in practice. At the same time, analysis of long-term trends is often useful and justified for predicting the most important changes in the market and making strategic decisions. The development of models relating to any single economic aspect of the product is often of no value because they do not take into account the impact of other elements of the product economics. However, price movement models have the advantage of reflecting in aggregate the effects of the myriad forces that determine supply and demand.

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